B.Sc Part II (Honrs)

Permutation group.

Def: Permutation :- Let A is a finite set having n distinct elements. Then a rule f which associates a one-one mapping of A onto itself is called a permutation of a set A.

If the number of distinct elements in the finite set A is n, then the permutation is of degree n.

i.e. if $f(A) \to A$ and f is one-one and onto then f is a permutation of degree n.

i.e. If $A = \{3, 4, 5, 6, 7\}$ is a finite set of five elements and $f(3) = 4$, $f(4) = 6$, $f(5) = 7$, $f(6) = 5$. and $f(7) = 3$, then we shall write

$$f = \begin{pmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 6 & 7 & 5 & 3 \end{pmatrix}$$

Here each element in the second row is the f-image of the element of the first row lying directly above it.

Theorem - Show that the set P_n of all permutations on n symbols is a finite group of order $\underline{|n}$ w.r.t. Composite of mappings as the operation. For $n \leq 2$, this group is Abelian and for $n > 2$, it is non-abelian.

Proof: - Let $A = \{x_1, x_2, x \text{ —— } x_n\}$ be a finite set having n distinct elements.

Let $f = \begin{pmatrix} x_1 & x_2 & x_3 & \text{———} & x_n \\ y_1 & y_2 & y_3 & \text{———} & y_n \end{pmatrix}$ be a permutation of degree n.

So a can be arrange in ${}^nP_n = \lfloor n$ ways and so there will be $\lfloor n$ distinct permutations of degree n.

Let P_n denote the set of all permutations of degree n, then P_n have $\lfloor n$ distinct element.

If $n=1$ then P_n is abelian: As every group of order 1 is Abelian.

If $n=2$ then $P_n = \lfloor 2 = 2$ elements and so P_n is abelian.

If $n>2$ then P_n is non-Abelian.

This is shown by following example

Let $f_1 = \begin{pmatrix} 1 & 2 & 3 & -- & n-1 & n \\ 2 & 3 & 4 & -- & n & 1 \end{pmatrix}$

and $f_2 = \begin{pmatrix} 1 & 2 & 3 & \text{——} & n-1 & n \\ 2 & 3 & 4 & \text{——} & n & 2 \end{pmatrix}$

Then $f_1 \cdot f_2 = \begin{pmatrix} 1 & 2 & 3 & \text{——} & n-1 & n \\ 1 & 3 & 4 & \text{——} & n & 2 \end{pmatrix}$

and $f_2 \cdot f_1 = \begin{pmatrix} 1 & 3 & 3 & 4 & \text{——} & n-1 & n \\ 3 & 2 & 4 & 5 & \text{——} & n & 1 \end{pmatrix}$

clearly $f_1 \cdot f_2 \neq f_2 \cdot f_1$. Hence P_n is non-Abelian if $n>2$.

—— Proved:

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